Joint and common cost allocation in a multi-level organization

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JOINT AND COMMON COST ALLOCATION IN A MULTI-LEVEL ORGANIZATION

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Abstract

A substantial part of the research project "The analysis of multi-level decisions" will be devoted to delegation within the firm, by coordinating instruments as transfer prices and budgets. To provide for the basic framework for this research, an integral model of the firm is to be developed, covering three issues, namely multiple technologies for intermediate and end products, "make-or-buy" decisions with respect to technical services, and common cost allocation due to the presence of general services.

This paper is devoted to the third of these issues. For the allocation of fixed overhead costs, a mathematical programming approach is proposed: by the appropriate use of optimal dual variables, the allocation does not distort the optimal product mix. The method also applies to a decomposable organization model.

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1. Introduction

In the accounting literature, a large number of articles is devoted to cost allocation problems. Extensive surveys and discussions concerning the subject can be found in [2] and [14]. As the contributions in the literature originate from accountants, economists and management scientists, there exists a great variety in assumptions, definitions, methodologies. This is, of course, not in favour of a systematic development of concepts and definitions.

The key problem in cost allocation is the occurrence of settings where a need for allocation arises although the particular cost is non-separable. Whether to identify this cost as "joint" or "common", is of secondary interest. In recent survey-like work, e.g. [2], [4] and [14], the "decision focus" is largely recommended, i.e. the analysis of cost allocation from an organizational point of view.

As part of the research project "Analysis of multi-level decisions", we are building a so-called integral model of the firm. The incorporation of cost allocation issues in this model will turn out to be most desirable.

The paper is organized as follows.
In chapter 2 we present the definitions of joint costs and common costs as applied in [2]. Chapter 3 treats a rather simple setting where cost allocation is required. The model and its solution should provide elementary insight in cost allocation problems. Then, in chapter 4, we will discuss the importance of the analysis with respect to organizational design. We conclude that chapter with a problem statement that reflects the relationship between multi-level decisions and joint/common cost allocation. Chapter 5 provides for a link with earlier work in the research project; the mathematical programming approach to be presented, may well be applied to the model of the firm as developed in [11] and also to decomposable organization models (e.g. [10]).

2. Cost allocation problems

2.1. Cost allocation

A most general indication of what is meant by a cost allocation is given in the following:

Definition 1:

A cost allocation is the efficient partitioning of a cost among a set of cost objects. The term "efficient" expresses that all of the cost is allocated, no more and no less.

In every cost allocation, three elements play a central role:
1. the total amount of costs to be allocated;
2. the cost objects among which the costs are to be allocated;
3. the allocation method or allocation basis that partitions the total cost.

The nature of the costs may be such that they are not entirely separable over the cost objects. Then a cost allocation problem arises due to the joint (common) nature of (part of) the costs.

2.2. Joint costs; common costs

Non-separable costs lead to allocation problems. The literature on this subject can broadly be divided into two classes, viz. "joint cost allocation" and "common cost allocation". In order to clarify this distinction, the terms "joint cost" and "common cost" will be defined.

Definition 2

A joint cost is a non-separable cost due to a non-separable production function which is defined on two or more products. Here the products are the cost objects.

So joint costs are related to joint production (see [5], chapter 9). After the split-off point, there exists separately identifiable products, which should jointly bear the costs incurred up to the split-off point (example: petroleum refinery).

Definition 3

A common cost is a non-separable cost due to a production function defined on just one intermediate product or service which is used in two or more divisions (or product lines). Here the divisions (or product lines) are the cost objects.

The divisions could have produced the intermediate product or service independently, but apparently they have decided to act together. Hence they jointly incur the costs of their joint action.

The reader may convince himself that these separate definitions do not always guarantee an unambiguous answer to the question whether some non-separable cost is either a member of the class of joint costs or of the class of common costs. But they provide some insight in the nature of non-separable costs.

2.3. Allocation problems

Because of our research focus (see chapter 4), we will not concentrate upon the differences between joint and common cost settings. What really is important, is the apparent non-separability of costs, caused by the absence of properties which enable some natural allocation among cost objects. Lacking this "natural allocation base", one has to design ("invent") an allocation rule. It is clear that every allocation rule is subject to a certain degree of arbitraryness.

Definition 4

A cost allocation problem consists of the design of an allocation rule (allocation method, allocation base) to be used for the allocation of certain non-separable (joint, common) costs.

3. The Louderback-Moriarity approach

From the various streams in the research on cost allocation problems, we will present one of the most elementary approaches, which is, though not really complicated, conceptually interesting.

3.1. The model

Consider a firm consisting of n divisions each of which has to produce a certain quantity of a (final) product. We will distinguish among three ways of producing the n products.
In the first production possibility, the divisions jointly buy an amount of raw material (costs: J), then extract n intermediate products, to be further processed by the divisions independently. A division finishing its own product incurs a cost equal to x_i. The total cost of this production possibility is thus: $J + x_1 + \ldots + x_n$.
Secondly, each division has the opportunity to buy the required quantity of its final product independently. This would cost: y_i, $i = 1,\ldots,n$.
One could also imagine that the finishing costs x_i of some division i are so low that this division is tempted to buy the same amount of raw material entirely for itself, and then extract and finish its product. The cost is then $J + x_i$ (as it is assumed impossible to buy smaller (and hence cheaper) amounts of the raw material).

It will be assumed that the first option, i.e., jointly buying the raw material and then further processing by individual divisions, is the cheapest alternative. Furthermore, every division has potential interest for this option as for every division the finishing cost is lower than the cost of buying the final product.

We now give a more formal presentation of the model.
Define:

n := number of divisions;

J := joint cost incurred if the divisions jointly buy the raw material;

x_i := i-th division's finishing cost, $i = 1,\ldots,n$;

y_i := i-th division's cost if it independently buys the required amount of its final product, $i = 1,\ldots,n$;

z_i := the cost of the i-th divisions next best alternative; since a division might be tempted to buy the raw material independently, it holds that $z_i = \min\{y_i, J+x_i\}$, $i = 1,\ldots,n$.

The two assumptions can be stated as follows:

Assumption 1: $J + x_1 + \ldots + x_n < y_1 + \ldots + y_n$

Assumption 2: for $i = 1,\ldots,n$, it holds that $x_i < y_i$.

For the firm as a whole, the alternative where all divisions jointly use the raw material is the most attractive one. This chapter is devoted to the design of an allocation rule which stimulates divisions to join in the common purchase and use of the raw material. Note that in this setting the common costs arise because of cost savings.

3.2. The allocation method and its properties

The model as described in section 3.1 gives rise to a cost allocation problem with respect to the joint costs for the common raw materials. Balachandran and Ramakrishnan [1] provide for a solution to this problem, mainly inspired by the contributions of Moriarity [12, 13], and Louderback [9]. The proposed allocation rule, to be referred to as the Louderback-Moriarity method, can be stated as follows.

Let:

T := the total cost to be allocated, i.e., $T = J + x_1 + \ldots + x_n$;

t_i := the cost allocated to division i, $i = 1,\ldots,n$.

Now the allocation rule is:

$$t_i = b_i + (T - \sum_{i=1}^{n} b_i) \frac{ptc_i}{\sum_{i=1}^{n} ptc_i},$$

where the basic charge b_i and the propensity to contribute ptc_i are taken as follows ($i = 1,\ldots,n$):

$$b_i = x_i \text{ (hence } T - \sum_{i=1}^{n} b_i = J)$$

$$ptc_i = z_i - x_i.$$

Indeed, all of the cost is allocated. Furthermore, this allocation rule has the five advantages as mentioned by Moriarity in his first article [12]. In short, these properties are:

1. the comparison of the costs of joint versus independent action is an essential element in the method (assumption 1);
2. $t_i \leq z_i$, so every division is encouraged to participate in the joint use of the raw material;
3. every cost object has a positive share in the savings of the firm as a whole;
4. every cost object bears a part of the total costs which is at least as much as its own costs x_i, in other words: no division is subsidized;
5. every division has an incentive to look for cheaper "next-best alternatives" z_i as that would reduce its ptc-fraction; the same holds with respect to the division's finishing costs x_i: finding cheaper finishing technologies results in a lower basic charge.

A last important property is that the allocation is in core. (This is proven in [1], p. 90). The core is a concept from game-theory which guarantees a form of stability; in effect, the above allocation rule being in core implies that it is untractable for groups of divisions to form a coalition in order to buy the amount of raw material for themselves only, and then further process it.

3.3. Evaluation

Balachandran and Ramakrishnan [1] conclude that the core criterion, which is a stability concept, generally does not uniquely determine a solution to the allocation problem. However, the "propensity-to-contribute" concept, which is responsible for the equity and fairness properties, selects exactly one allocation rule in the core. So the economic principle of stability together with the accounting principles equity and fairness lead to a unique solution of the allocation problem. The authors of [1] note that the allowance for coalitions, to be formed by subsets of divisions, is conceptually more attractive, but will cause increased computational difficulties.

What we find interesting about the propensity-to-contribute concept, is the observation of having a margin between a division's basic charge and the cost of its next-best alternative. The fairness property apparently implies the allocation of a fraction of the cost savings which is proportional to the division's margin z_i-b_i. Of course, this meaning of fairness is more or less arbitrary: the division's margin could have been utilized in a different way.

Although we have first presented the rule and then cited its properties, it is clear that in the solution of allocation problems, the statement of a set of desirable properties will be an essential element.

4. Cost allocation and multi-level decisions

In chapter 2 we have discussed terms like joint costs and common costs. Then we illustrated these concepts with a particular approach from the literature on cost allocation problems. Now we will discuss the need for a thorough analysis of cost allocation from an organizational design point of view. In this light two recent contributions are treated. Finally, we formulate the problem in view of the research project "The analysis of multi-level decisions".

4.1. Motives for the analysis of allocation problems

A firm faces allocation problems whenever joint or common costs are incurred. Recalling the organizational setting in the Louderback-Moriarity approach, we observe that common costs often arise because of cost savings due to joint action (instead of independent behaviour of the cost objects). A second source of joint costs is the occurrence of internal "general services" like Central Management, Research and Development, Public Relations. These departments produce common goods from which all other subunits in the firm benefit, thereby giving lead to common costs to be beared by the firm as a whole. Finally, we observe that every allocation of a fixed cost (e.g., factory overhead, depreciation of machinery) is implicitly a joint cost allocation.

From a global point of view, i.e. if some overall optimization model for the entire firm is applied, it is often concluded that allocations should not be taken into account. Discussions and more references on this theoretical observation can be found in [2, pp. 7-8] and [14, p. 5]. Briefly speaking, the literature suggests "allocation free" decision models. But, at the same time, we observe that in any organization of a reasonable size a certain degree of decentralization has taken place. Decentralized information and delegation of decision-making authority are the key words in the attempt to reduce the complexity of decision-making in a large firm (see [8], chapter 14 for an excellent treatment of these issues). This implies that the use of overall optimization problems as models for a decentralized firm is highly unrealistic. As a consequence, different type of models, that explicitly recognize the decentralization features, are required. It is not a-priorily clear whether these models again turn out to be allocation free.
Summarizing, the theoretical justification for not considering cost allocation may well be non-valid in more decentralized settings.

The potential theoretical improvements by the explicit recognition of allocation issues in modelling complex, decentralized organizations can be of practical significance, as, not only by definition (see chapter 2), common cost allocations do occur in the context of multi-division firms. The third and probably most important motive, which is again "decentralization-inspired", is that, whenever a process of delegation of decision authority occurs, the division managers become local decision-makers with their own responsibilities, goals, preferences. Now a motivation and a coordination problem arises, as Dopuch et. al [5], p. 330, note. How can local decision-makers be lead towards firm-wide optimal decisions? Thomas ([14], pp. 7-8) provides reasonable arguments why a division manager's decisions might be affected by allocated costs. As an example, we return to the Louderback-Moriarity approach. Property 2. and 3. (see section 3.2) are clearly stimulating divisions to joint action; property 5. implicitly assumes that the cost allocation has an effect on a division manager's behaviour.

4.2. Cost allocation in a decentralized organization structure

Altogether, in a multi-division setting, common cost allocation is not only likely to occur, but will have an effect on decision-making within the firm. This provides a strong incentive to search for allocation rules which are consistent with the organizational structure. Therefore, we discuss two recent contributions in the literature that incorporate organizational considerations.

Zimmerman [15] is, to the author's perception, the first one who observes that "cost allocations, managerial behaviour, and the structure of the organization, including the incentives facing the managers, are extricably linked". In his article, Zimmerman states that in certain situations cost allocations yield positive net benefits to the firm. This notion is further explored by indicating the relationship with the agency problem. It is shown that cost allocations can act as a lump-sum tax which reduces a manager's consumption of perquisites.

A second component of the analysis refers to allocation rules where allocated costs are coupled to a division's use of production factors (e.g. labour). Similar rules, which are sometimes observed in practice, might induce divisions to switch to labour extensive production technologies, which in turn can be sub-optimal from a firm-wide point of view.

Contrary to Zimmerman's set up, which is of a more or less verbal, introductory kind, Cohen and Loeb [3] provide for a conceptual approach to common cost allocation in a divisionalized firm. They start with the characterization of a pure common good: once produced, it is free for all divisions to consume. (Example: corporate image advertising.) The opposite of a pure common good is a pure private good: once consumed by some division, it is not available to other divisions anymore. (Example: collective typing service department in a university.) Whereas Hughes and Scheiner [6, p. 90] proved that there does not exist a full cost allocation scheme for pure private goods which enhances "efficient decentralization", Cohen and Loeb show "that it is possible to reach an efficient allocation by decentralization and fully allocating costs". Their organizational model consists of a number of divisions plus a corporate headquarters. The divisions all require a certain pure common input, which is to delivered by headquarters. The provision of the common input leads to common costs, incurred by headquarters, and to be allocated to the divisions, which are considered as profit centers. Having incomplete information with respect to the divisional profit functions, it is difficult for headquarters to determine the right level of the common input. The allocations of the common costs should generate information about divisional demand and thus be helpful in choosing the optimal level of the common input.

It is not appropriate to present the complete model as developed in [3]. The main result is that the divisions are charged according to the marginal benefits they receive from the common input. This brings about the "free-rider problem": divisions have an incentive to understate their demands and still enjoy the benefits of the common good. Of course, this drawback is intimately related to the asymmetric information structure.

4.3. Contribution to the research project "The analysis of multi-level decisions"

We have described motives for and contributions to the organizational design approach to cost allocation problems. It is now logical that within the research project "Analysis of multi-level decisions", with decentralization, delegation and coordination as main topics, profound attention will be paid to cost allocation. In effect, we will build in this topic in the development of what is called an "integral model of the firm".

The integral model of the firm should provide for a basic framework for the analysis of multi-level issues. In previous work we presented a model for the firm including multiple technologies for products and "make-or-buy" decisions concerning technical services. Though we started in a (Leontief) input-output setting, we ended up with a mixed-integer programming formulation (cf. [11]). In casual input-output analysis, the (usually fixed) costs of the general services are allocated among the other sub-units in the firm. As an extension to the model developed earlier (in [11]), we will assume that, besides end products, intermediate products and technical services producing sub-units, the firm also contains a "General Services" sector. The costs incurred by general services departments may be variable in the long run, but, in the short run, they are assumed to be entirely fixed.
The problem to be analyzed is then:
"Can cost allocations be used as coordinating instruments that direct the sub-

units' managers towards the correct, i.e., firm-wide optimal, choices of technologies and make-or-buy decisions?"

5. Fixed overhead allocation

The first half of this paper can be seen as an introduction in the field of joint/common costs. Then, in chapter 4, we have directed the attention to the multi-level decision-making aspects of allocation problems. Section 4.3 was concluded with a problem statement which asks for the incorporation of cost allocation issues in the decision model as developed in earlier work, i.e. [11]. In this chapter we introduce the mathematical programming approach to common cost allocation. The analysis is rooted in an article by Kaplan and Thompson ([7], 1971). These authors describe how a fixed overhead cost can be allocated among activities in the context of a linear programming model of the firm, without distorting the relative profitability of products. Because of the resemblance with the decision model in [11], the work of Kaplan and Thompson is closely related to our problem statement. Secondly, their method also applies to decomposable organisation models.

5.1. The basic theorem

Consider the LP-problem:

$$\begin{array}{ll} \text{Maximize} & p\,x \\ \text{st.} & A_1\, x \leq a_1 \\ & \vdots \\ & A_n\, x \leq a_n \\ & \quad x \geq 0 \end{array} \tag{5.1}$$

The (finite) optimum solution value P is attained at $x = \bar{x}$, so $P = p\bar{x}$. The associated dual problem is:

$$\begin{array}{ll} \text{Minimize} & u_1a_1 + \ldots + u_na_n \\ \text{s.t.} & u_1A_1 + \ldots + u_nA_n \geq p \\ & u_1,\ldots,u_n \geq 0 \end{array} \tag{5.2}$$

with optimal solution $(\bar{u}_1,\ldots,\bar{u}_n)$, so $P = \bar{u}_1a_1 + \ldots + \bar{u}_na_n$.
Let k_i $(i = 1,\ldots,n)$ such that $k_i \geq -1$. We present a "perturbed" version of (5.1):

$$\begin{array}{ll} \text{Maximize} & (p + \sum_{i=1}^{n} k_i\bar{u}_iA_i)y \\ \text{s.t.} & A_1\, y \leq a_1 \\ & \vdots \\ & A_n\, y \leq a_n \\ & y \geq 0. \end{array} \tag{5.3}$$

Theorem 1

If we take $\bar{y} = \bar{x}$, then $\bar{y}$ is an optimal solution to (5.3).
Secondly, $((1+k_1)\bar{u}_1,\ldots,(1+k_n)\bar{u}_n)$ is an optimal dual solution.
The optimum solution value is $P + \sum_{i=1}^{n} k_i\,\bar{u}_i\,a_i$.

The proof of this theorem, which exploits the complementary slack conditions with respect to the original problem (5.1)-(5.2), is given in appendix A.

Taking $k_1 = \ldots = k_n = k$, $-1 \leq k \leq 0$, we obtain the result of Kaplan and Thompson ([7], p. 356). Their version of (5.1) is simply:

$$\begin{array}{ll} \text{Maximize} & p\,x \\ \text{s.t.} & A\,x \leq b \\ & x \geq 0 \end{array} \tag{5.4}$$

They take $k = -\frac{H}{P}$ where H is some fixed overhead cost, $0 \leq H \leq P$. The solution of the perturbed version of (5.4), i.e.

$$\begin{array}{lll} \text{Maximize} & (p + k\bar{u}\,A)x & \\ \text{s.t.} & A\,x \leq a & \\ & x \geq 0 & \end{array} \qquad (5.5)$$

is again the optimal $\bar{x}$ of (5.4), with solution value $P + k\bar{u}a = P - H$. So all overhead is allocated without distortion of the optimal product mix.

5.2. Alternative formulation

It is not difficult to state the analogon of theorem 1 which applies to a minimization formulation. We now give the result:

Corollary 1

Consider the LP-problem:

$$\begin{array}{lll} \text{Minimize} & c\,x & \\ \text{s.t.} & A_i\,x \geq a_i, & i = 1,\ldots,n \\ & x \geq 0 & \end{array} \qquad (5.6)$$

Let the finite optimum solution value P be attained at $x = \bar{x}$.
Let $(\bar{u}_1,\ldots,\bar{u}_n)$ be an optimal dual solution. The perturbed problem:

$$\text{Minimize} \quad (c + \sum_{i=1}^{n} k_i\,\bar{u}_i\,A_i)y$$

$$\begin{array}{lll} \text{s.t.} & A_i\,y \geq a_i, & i = 1,\ldots,n \\ & y \geq 0 & \end{array}$$

where $k_i \geq -1$ $(i = 1,\ldots,n)$, has $y = \bar{x}$ as an optimal solution.
Furthermore, $((1+k_1)\bar{u}_1,\ldots,(1+k_n)\bar{u}_n)$ is dual optimal.
The optimum solution value is $P + \sum_{i=1}^{n} k_i\,\bar{u}_i\,a_i$. []

5.3. Application to the integral model of the firm

This result can be very useful for the decision model developed in [11]. We now highlight two main opportunities.
Firstly, the input-output based model including multiple technologies for products, appeared to be a (continuous) LP-problem of the form (5.6), say:

$$\begin{aligned} &\text{Minimize } c\,x \\ &\text{s.t.} \quad A\,x \geq a \\ &\qquad\quad\ \ x \geq 0 \end{aligned} \tag{5.7}$$

Let $\bar{x}$ be the optimal solution. Suppose we want to allocate a fixed cost H among the activities, so among the technologies in the end product and intermediate product sector (for terminology, see [11], chapter 2). Hereto, we define $k := H/(c\bar{x})$. The following modified LP-problem still has $\bar{x}$ as an optimum solution:

$$\begin{aligned} &\text{Minimize } (c + k\bar{u}\,A)x \\ &\text{s.t.} \qquad A\,x \geq a \\ &\qquad\qquad\ \ x \geq 0 \end{aligned} \tag{5.8}$$

where $\bar{u}$ is an optimal dual solution to (5.7). The optimum solution value is $c\bar{x} + H$, so without distortion of the optimal activity mix, the fixed cost is allocated!

Secondly, Kaplan and Thompson [7] suggest an extension of their method for the case of a mixed-integer programming formulation. In our previous report [11], this type of modelling has been chosen to incorporate the "make-or-buy" issue (viz. by introducing 0-1 variables). We expect to find a modification of corollary 1 similar to the just mentioned extension in [7].

Summarizing, we have at hand a potentially powerful "tool", i.e., corallary 1, for the incorporation of common cost allocation in the earlier developed model in [11]. The precise mathematical analysis and economic interpretation of the outlined ideas form a topic of further research to be reported on in a future report.

5.4. Application to decomposable organization models

This section is somewhat independent of the rest of this paper. On the other hand, it is included in this chapter, as it is concerned with the allocation of a fixed cost in the context of a decomposable model of the firm. Moreover, the proposed method is a direct application of theorem 1.

The model (cf [10], chapter 2)

Consider the following block-angular, and hence decomposable, LP-problem as the model of a divisionalized firm:

$$\begin{array}{lllll} \text{Maximize} & c_1x_1 + \dots + c_nx_n & & & \\ \text{s.t.} & A_1x_1 + \dots + A_nx_n & \leq a & & \\ & B_1x_1 & \leq b_1 & & \\ & & \vdots & & \qquad (5.9) \\ & \qquad\qquad B_nx_n & \leq b_n & & \\ & \quad x_1,\dots,x_n & \geq 0 & & \end{array}$$

The blocks $B_ix_i \leq b_i$ represent the local constraints of the i-th division ($i = 1,\dots,n$); the common rows $\Sigma A_ix_i \leq a_i$ reflect the common use of certain scarce resources to be dividend among the divisions by the central unit.

Implementation of optimal plan

Suppose that a planning procedure à la Dantzig-Wolfe or à la Benders ([10], chapter 4) is applied to solve (5.9). In the course of the last iteration of such a procedure, the central unit learns the amounts of scarce resources $\bar{a}_1,\dots,\bar{a}_n$ ($\bar{a}_i \geq 0$, $\Sigma\, \bar{a}_i = a$) to be allocated to the divisions, plus the optimal shadow price $\bar{u}_0$ of these resources. Now the central unit wants to implement the optimal plan as found in the planning procedure ([10], section 4.1.3, 4.2.3), while allocating a fixed cost H. It is assumed that $H \leq \bar{u}_0\, a$. Define $k := H/(\bar{u}_0 a)$.

Theorem 2

Consider the n problems ($i = 1,\dots,n$):

$$\begin{aligned}
\text{Maximize} \quad & (c_i - k\bar{u}_0 A_i)y_i \\
\text{s.t.} \quad & A_i y_i \leq \bar{a}_i \\
& B_i y_i \leq b_i \\
& y_i \geq 0
\end{aligned} \qquad (5.10)$$

Let $(\hat{x}_1,\dots,\hat{x}_n)$ be an optimal solution to the original problem (5.9) with (finite) objective function value P. Then $y_i = \hat{x}_i$ is optimal to (5.10) with objective function value $c_i \hat{x}_i - k\bar{u}_0\bar{a}_i$ ($i = 1,\dots,n$).
The sum of these n optimum solution values is P-H.

The proof of this theorem, which is based on theorem 1 and certain decomposition features, is given in Appendix B.

Allocation rule

Let each division solve the problem

$$\begin{aligned}
\text{Minimize} \quad & (c_i - k\bar{u}_0 A_i)x_i \\
\text{s.t.} \quad & A_i x_i \leq \bar{a}_i \\
& B_i x_i \leq b_i \\
& x_i \geq 0
\end{aligned}$$

Economic interpretation

The original "overall" problem (5.9) can be seen as the problem of finding those activity levels, $x_1,\dots,x_n$ that maximize the gross profit of the firm. This planning problem can be solved by a procedure à la Dantzig-Wolfe or à la Benders, thereby taking into account the decentralized information structure [10, p. 34]. At the end, this yields the optimal amounts of common resources for each division (i.e. $\bar{a}_1,\dots,\bar{a}_n$) and also the optimal shadow price $\bar{u}_0$ of these resources.

In order to implement the optimal plan, each division is given in its part $\bar{a}_i$ of the common resources. Furthermore, the division pays a price for the actual use (i.e. $A_i x_i$) of common resources which is a fraction of the optimal shadow price. This fraction k is $k = H/(\bar{u}_0 a)$. The price for common resources, $k\bar{u}_0$, has two properties, viz:

1. the final activity levels as computed and implemented by the divisions still realize the same gross profit (say P);
2. all of the overhead cost H is allocated.

Now 1. and 2. imply that the net profit of the firm is maximized. (It is equal to P-H.)

A few important remarks are in order.

The cost H is assumed fixed, and hence independent of the x_i, so the divisions cannot influence the level of H. If H would have been the cost of the general services department, this means that the divisions cannot influence the effort level of the general services department. The allocation rule does not provide some means of evaluation of "General Services".

The fraction k is uniform over divisions and (common) restrictions. This is not a necessary condition for the allocation method to succeed. E.g. if the total valuation of just one of the common resources solely exceeds the overhead cost, then the overhead can already be covered if divisions are only charged for the use of that particular common resource.

Finally, recall the assumption $H \leq u_0\bar{a}$. If this condition is not fulfilled, the proposed allocation rule cannot be directly used.

6. Summary

The main points of this paper are:

1) Joint/common cost allocation problems arise whenever non-separable costs are incurred to be allocated to products or divisions. Definitions and an illustration of related concepts are given in chapters 2 and 3, respectively.

2) It is highly desirable to put emphasis on the underlying reasons for cost allocation, in particular, the link with the organizational structure and managerial behaviour. As a consequence, cost allocation issues deserve substantial attention within the "Analysis of multi-level decisions" (ch. 4).

3) As a completion to earlier work concerning the integral model of the firm [11], we extended the model as developed there with a sector "General Services". In chapter 5, a promising approach to the allocation of the fixed costs of general services departments over other sub-units is outlined.

4) The mathematical programming approach in chapter 5 also applies to decomposable organization models (e.g., [10]).

The ideas as referred to under 3) and 4) will be subject to a thorough mathematical analysis and economic interpretation to be reported on in a future report.

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Appendix A: Proof of theorem 1

The conjectures to be proven are:

1. $y = \bar{x}$ is an optimal solution to (5.3), where $\bar{x}$ is an optimal solution to (5.1).
2. $((1+k_1)\bar{u}_1,\ldots,(1+k_n)\bar{u}_n)$ is an optimal solution to the dual of (5.3).
3. The optimum solution value is $P + \sum_{i=1}^{n} k_i \bar{u}_i a_i$.

Of course, $\bar{y}$ is a feasible solution of problem (5.3). We now show that $((1+k_1)\bar{u}_1,\ldots,(1+k_n)\bar{u}_n)$ is a feasible solution to the dual of (5.3), i.e. to

$$\begin{aligned}
\text{Minimize}\quad & v_1 a_1 + \ldots + v_n a_n \\
\text{s.t.}\quad & v_1 A_1 + \ldots + v_n A_n \geq p + \sum_{i=1}^{n} k_i \bar{u}_i A_i \\
& v_1,\ldots,v_n \geq 0
\end{aligned}$$

Substitution of $v_i = (1+k_i)\bar{u}_i \quad (i = 1,\ldots,n)$ yields:

$$\begin{aligned}
\sum_{i=1}^{n} (1+k_i)\bar{u}_i A_i &= \sum_{i=1}^{n} \bar{u}_i A_i + \sum_{i=1}^{n} k_i \bar{u}_i A_i \\
&\geq p + \sum_{i=1}^{n} k_i \bar{u}_i A_i .
\end{aligned}$$

As $k_i \geq -1$, every $(1+k_i)\bar{u}_i$ is ≥ 0.

Secondly, $\bar{y}$ and $((1+k_1)\bar{u}_1,\ldots,(1+k_n)\bar{u}_n)$ have the same objective function value:

$$\begin{aligned}
(p + \sum_{i=1}^{n} k_i \bar{u}_i A_i)\bar{y} &= p\,\bar{x} + \sum_{i=1}^{n} k_i \bar{u}_i A_i \bar{y} \\
&= P + \sum_{i=1}^{n} k_i \bar{u}_i a_i
\end{aligned} \tag{A.1}$$

due to the complementary slackness conditions as applied to (5.1)-(5.2).
The dual solution value is:

$$\sum_{i=1}^{n} (1+k_i)\bar{u}_i\ a_i = \sum_{i=1}^{n} \bar{u}_i\ a_i + \sum_{i=1}^{n} k_i\ \bar{u}_i\ a_i$$

$$= P + \sum_{i=1}^{n} k_i\ \bar{u}_i\ a_i \tag{A.2}$$

So we have shown that $\bar{y}$ and $((1+k_i)\bar{u}_1,\ldots,(1+k_n)\bar{u}_n)$ are primal-dual feasible, with equal objective function value. But then this pair is primal-dual optimal. The optimum solution value follows from (A.1) (or A.2)). □

Appendix B: Proof of theorem 2

Let $\bar{u}_0$ be the optimal shadow price of the scarce resources. In the Dantzig-Wolfe as well as the Benders' case, it is possible to prove the existence of $\tilde{u}_1,\ldots,\tilde{u}_n$ such that $(\bar{u}_0, \tilde{u}_1,\ldots,\tilde{u}_n)$ is an optimal solution to the dual of (5.9). From this observation it is easy to prove that $(\bar{u}_0,\ldots,\bar{u}_0,\tilde{u}_1,\tilde{u}_2,\ldots,\tilde{u}_n)$ is an optimal solution to the dual of:

$$\begin{array}{lllll} \text{Maximize} & c_1x_1 + \cdots + c_nx_n & & & \\ \text{s.t.} & A_1x_1 & & \leq \bar{a}_1 & \\ & & \ddots & \vdots & \\ & & A_nx_n & \leq \bar{a}_n & \\ & B_1x_1 & & \leq b_1 & \\ & & \ddots & \vdots & \\ & & B_nx_n & \leq b_n & \\ & & x_1,\ldots,x_n & \geq 0 & \end{array} \tag{B.1}$$

Let $\hat{x}_1,\ldots,\hat{x}_n$ be an optimal solution of problem (B.1).
Now we can apply theorem 1. Introduce scalars k_i with $k_i \geq -1$, $i = 1,\ldots,n$. The following modified version of (B.1), i.e.:

$$\begin{array}{lllll} \text{Maximize} & (c_1+k_1\bar{u}_0A_1)y_1 + \cdots + (c_n+k_n\bar{u}_nA_n)y_n & & & \\ \text{s.t.} & A_1\,y_1 & & \leq \bar{a}_1 & \\ & & \ddots & \vdots & \\ & & A_n\,y_n & \leq \bar{a}_n & \\ & B_1\,y_1 & & \leq b_1 & \\ & & \ddots & \vdots & \\ & & B_n\,y_n & \leq b_n & \\ & & y_1,\ldots,y_n & \geq 0 & \end{array} \tag{B.2}$$

still has $\hat{x}_1,\ldots,\hat{x}_n$ as an optimal solution. The optimum solution value is

$$\sum_{i=1}^{n} c_i\,\hat{x}_i + \sum_{i=1}^{n} k_i\,\bar{u}_0\,\bar{a}_i.$$

Observe that (B.2) is composed of n independent problems of the form:

$$\begin{array}{lll} \text{Maximize} & (c_i + k_i\bar{u}_0 A_i)y_i & \\ \text{s.t.} & A_i\, y_i \le \bar{a}_i & \quad (B.3) \\ & B_i\, y_i \le b_i & \\ & y_i \ge 0 & \end{array}$$

Obviously, $\hat{x}_i$ is an optimal solution to (B.3) with objective function value $c_i\hat{x}_i + k_i\bar{u}_0\bar{a}_i$.

If we take all $k_i = -H/(\bar{u}_0 a)$, problem (B.3) is exactly problem (5.10)! The sum of the n optimum solution values is

$$\sum_{i=1}^{n} c_i\hat{x}_i - \sum_{i=1}^{n} k\bar{u}_0\bar{a}_i = P-H$$

as $\bar{a}_1 + \ldots + \bar{a}_n = a$ and $k = H/(\bar{u}_0 a)$. □

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